A Compact 2-D Full-Wave Finite-Difference Frequency-Domain Method for General Guided Wave Structures

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***Abstract*—A compact two-dimensional (2-D) full-wave finite-difference frequency-domain method is proposed for the analysis of dispersion characteristics of a general guided wave structure. Because the longitudinal field components are eliminated in the proposed method, only four transverse field components are involved in the final resulting eigen equation. This feature considerably reduces the required CPU time as compared to the existing approaches by which six field components are comprised. Additionally, unlike other 2-D finite-difference schemes that determine the eigenfrequency for a given propagation constant, the new method finds the propagation constant β for a given k_0 (frequency). The new method has been verified by examining the computed results of a number of typical guided wave structures with the published results. Very good agreement is achieved.**

***Index Terms*—Anisotropic media, eigenvalues and eigenfunctions, finite-difference methods, frequency-domain analysis, waveguide.**

I. Introduction

The determination of the dispersion characteristics of a general guided wave structure is a very important subject in practical engineering designs. Numerous full-wave techniques tackling the problems have been proposed by many researchers [1]–[8]. For example, the approach used to determine the characteristics in [1] is based on a three-dimensional (3-D) finite-difference time-domain (FDTD) simulation. Because the method involves a 3-D mesh, large memory space and long CPU time are required. To alleviate these problems, Xiao *et al.* introduced a two-dimensional (2-D) FDTD approach that uses only a 2-D mesh consisting of all six field components [2]. In the approach, the propagation constant is given as an input parameter for solving for eigenfrequencies. A further step was taken to reduce the grid to a truly 2-D grid by approaching the mesh size in the propagation direction to zero [4]. To avoid dealing with complex variables and to improve the efficiency, a variable transformation was introduced that leads to a real-variable algorithm [5]. Hong and Park applied the real-variable 2-D compact FDTD to the analysis of the dispersion characteristic of a unilateral fin-line in [6].

Although the above-mentioned 2-D FDTD approaches have the advantages of CPU time and memory saving over those 3-D methods, they all need to give the propagation constant β as an input parameter and have to find the eigenfrequencies of interest via discrete Fourier transform. To cope with the predicament, a novel 2-D finite-difference frequency-domain (FDFD) algorithm was proposed recently [7], in which the propagation constant is sought for a given frequency. However, all the six field components need to be employed to yield an eigen equation.

In this paper, a compact 2-D full-wave FDFD method is proposed for solving for propagation constant β of a general transmission line for a given k_0 (frequency). To convert a physical guided wave problem into an eigen problem, only four transverse field components are involved. The resultant eigen equation is constructed by a highly sparse matrix. In contrast to the existing approaches that use six field components, the new method is much more efficient in computation. Needless to say that the feature of finding β for a given k_0 is more pertinent in practical applications.

II. Theory

It is assumed that the guided wave structure is uniform along axis z and the wave propagates in the positive z direction. The fields in a general guided wave structure can be expressed as

$$\vec{E}(x, y, z) = [E_x(x, y)\hat{x} + E_y(x, y)\hat{y} + E_z(x, y)\hat{z}]e^{-j\beta z} \tag{1}$$

$$\vec{H}(x, y, z) = [H_x(x, y)\hat{x} + H_y(x, y)\hat{y} + H_z(x, y)\hat{z}]e^{-j\beta z} \tag{2}$$

where the time variation of $e^{j\omega t}$ is suppressed. Therefore, the derivatives with respect to z can be replaced by $\partial/\partial z = -j\beta$.

Although the approach proposed is applicable to a general anisotropic type of transmission line, for the simplicity of illustration, the derivation below will be limited to the case of electric anisotropic media with diagonal dielectric constant tensor $\overline{\overline{\varepsilon_r}}$. By normalizing field components with square root of the free-space wave impedance such that $\vec{H}' = \vec{H} \cdot \sqrt{\eta_0}$ and $\vec{E}' = \vec{E}/\sqrt{\eta_0}$, Maxwell's curl equations become

$$-jk_0\vec{H} = \nabla \times \vec{E} \quad jk_0\overline{\overline{\varepsilon_r}} \cdot \vec{E} = \nabla \times \vec{H} \tag{3}$$

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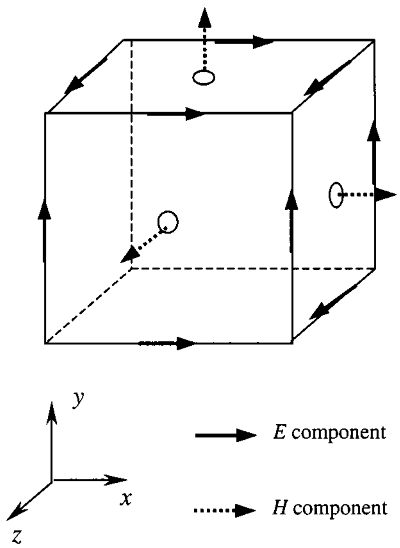

Fig. 1. Conventional Yee's 3-D lattice.

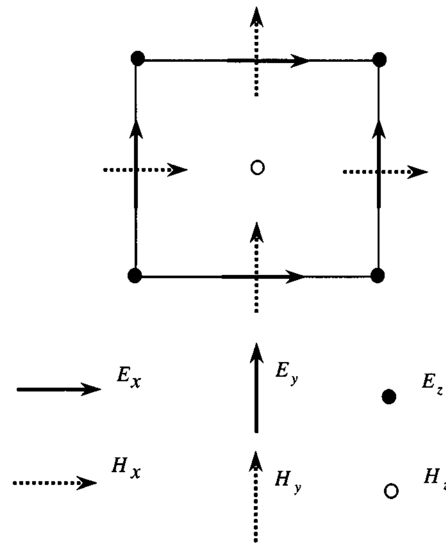

Fig. 2. Compact 2-D lattice.

where the prime sign is omitted for the sake of simplicity.

With Yee's grid system shown in Fig. 1, (3) can be discretized as

$$\begin{cases} H_x(i,j) = j\,\dfrac{1}{k_0h_y}\,[E_z(i,j+1)-E_z(i,j)] \\ \qquad\qquad -\dfrac{\beta}{k_0}\,e^{-j\Delta}E_y(i,j) \\ H_y(i,j) = \dfrac{\beta}{k_0}\,e^{-j\Delta}E_x(i,j) \\ \qquad\qquad -j\,\dfrac{1}{k_0h_x}\,[E_z(i+1,j)-E_z(i,j)] \\ H_z(i,j) = j\,\dfrac{1}{k_0h_x}\,[E_y(i+1,j)-E_y(i,j)] \\ \qquad\qquad -j\,\dfrac{1}{k_0h_y}\,[E_x(i,j+1)-E_x(i,j)] \end{cases} \tag{4}$$

and

$$\begin{cases} \varepsilon_{xx}E_x(i,j) = -j\,\dfrac{1}{k_0h_y}\,[H_z(i,j)-H_z(i,j-1)] \\ \qquad\qquad +\dfrac{\beta}{k_0}\,e^{-j\Delta}H_y(i,j) \\ \varepsilon_{yy}E_y(i,j) = j\,\dfrac{1}{k_0h_x}\,[H_z(i,j)-H_z(i-1,j)] \\ \qquad\qquad -\dfrac{\beta}{k_0}\,e^{-j\Delta}H_x(i,j) \\ \varepsilon_{zz}E_z(i,j) = -j\,\dfrac{1}{k_0h_x}\,[H_y(i,j)-H_y(i-1,j)] \\ \qquad\qquad +j\,\dfrac{1}{k_0h_y}\,[H_x(i,j)-H_x(i,j-1)] \end{cases} \tag{5}$$

where h_x, h_y and h_z are mesh sizes in x, y, and z directions, respectively, and $\Delta = \beta h_z/2$.

As h_z approaches zero, the Yee's grid is reduced to a compact 2-D grid as shown in Fig. 2. Consequently, (4) and (5) become the following 2-D form:

$$\begin{cases} H_x(i,j) = j\,\dfrac{1}{k_0h_y}\,[E_z(i,j+1)-E_z(i,j)]-\dfrac{\beta}{k_0}\,E_y(i,j) \\ H_y(i,j) = \dfrac{\beta}{k_0}\,E_x(i,j) \\ \qquad\qquad -j\,\dfrac{1}{k_0h_x}\,[E_z(i+1,j)-E_z(i,j)] \\ H_z(i,j) = j\,\dfrac{1}{k_0h_x}\,[E_y(i+1,j)-E_y(i,j)] \\ \qquad\qquad -j\,\dfrac{1}{k_0h_y}\,[E_x(i,j+1)-E_x(i,j)] \end{cases} \tag{6}$$

and

$$\begin{cases} \varepsilon_{xx}E_x(i,j) = -j\,\dfrac{1}{k_0h_y}\,[H_z(i,j)-H_z(i,j-1)] \\ \qquad\qquad +\dfrac{\beta}{k_0}\,H_y(i,j) \\ \varepsilon_{yy}E_y(i,j) = j\,\dfrac{1}{k_0h_x}\,[H_z(i,j)-H_z(i-1,j)] \\ \qquad\qquad -\dfrac{\beta}{k_0}\,H_x(i,j) \\ \varepsilon_{zz}E_z(i,j) = -j\,\dfrac{1}{k_0h_x}\,[H_y(i,j)-H_y(i-1,j)] \\ \qquad\qquad +j\,\dfrac{1}{k_0h_y}\,[H_x(i,j)-H_x(i,j-1)]. \end{cases} \tag{7}$$

Eliminating E_z and H_z from (6) and (7), we obtain

$$\begin{aligned} &\frac{\beta}{k_0}\,E_x(i,j) \\ &= -\frac{1}{k_0^2\varepsilon_{zz}h_xh_y}\,[H_x(i,j-1)-H_x(i+1,j-1) \\ &\qquad - H_x(i,j)+H_x(i+1,j)] \\ &\quad + \frac{1}{k_0^2\varepsilon_{zz}h_x^2}\,H_y(i-1,j)+\left(1-\frac{2}{k_0^2\varepsilon_{zz}h_x^2}\right)H_y(i,j) \\ &\quad + \frac{1}{k_0^2\varepsilon_{zz}h_x^2}\,H_y(i+1,j) \end{aligned} \tag{8}$$

$$\frac{\beta}{k_0} E_y(i, j) = -\frac{1}{k_0^2 \varepsilon_{zz} h_y^2} H_x(i, j-1) - \left(1 - \frac{2}{k_0^2 \varepsilon_{zz} h_y^2}\right) H_x(i, j) - \frac{1}{k_0^2 \varepsilon_{zz} h_y^2} H_x(i, j+1) + \frac{1}{k_0^2 \varepsilon_{zz} h_x h_y} \cdot [H_y(i-1, j) - H_y(i, j) - H_y(i-1, j+1) + H_y(i, j+1)] \tag{9}$$

$$\frac{\beta}{k_0} H_x(i, j) = \frac{1}{k_0^2 h_x h_y} [E_x(i-1, j) - E_x(i, j) - E_x(i-1, j+1) + E_x(i, j+1)] - \frac{1}{k_0^2 h_x^2} E_y(i-1, j) - \left(\varepsilon_{yy} - \frac{2}{k_0^2 h_x^2}\right) E_y(i, j) - \frac{1}{k_0^2 h_x^2} H_y(i+1, j) \tag{10}$$

$$\frac{\beta}{k_0} H_y(i, j) = \frac{1}{k_0^2 h_y^2} E_x(i, j-1) + \left(\varepsilon_{xx} - \frac{2}{k_0^2 h_y^2}\right) E_x(i, j) + \frac{1}{k_0^2 h_y^2} E_x(i, j+1) - \frac{1}{k_0^2 h_x h_y} \cdot [E_y(i, j-1) - E_y(i+1, j-1) - E_y(i, j) + E_y(i+1, j)]. \tag{11}$$

Because the longitudinal components have been eliminated in the final equations, the boundary condition $E_z = 0$ on the surface of conductors has to be reflected indirectly. It can be done by inserting the boundary condition into (6) and (7) and systematically modifying (8)–(11) accordingly. For example, for $E_z(i+1, j) = 0$ in (6), (8) will take the form of

$$\frac{\beta}{k_0} E_x(i, j) = -\frac{1}{k_0^2 \varepsilon_{zz} h_x h_y} [H_x(i, j-1) - H_x(i, j)] + \frac{1}{k_0^2 \varepsilon_{zz} h_x^2} H_y(i-1, j) + \left(1 - \frac{1}{k_0^2 \varepsilon_{zz} h_x^2}\right) H_y(i, j). \tag{12}$$

As in FDTD, the continuity condition of electric fields across two dielectric mediums is ensured by setting the dielectric constant on the interface as the average of the dielectric constants in the two regions.

After implementing all the boundary conditions, (8)–(11) can be concluded as an eigen problem as

$$[A] \cdot \{x\} = \lambda \{x\} \tag{13}$$

where $[A]$ is the right-hand sparse matrix coefficient listed in (8)–(11), $\lambda = \beta/k_0$, and $\{x\} = \{E_x, E_y, H_x, H_y\}^T$. The conductor boundary conditions for transverse electric field components can be applied straightforwardly. For the convenience of applying the boundary conditions for the transverse electric field components, (13) can be rewritten as a generalized eigenvalue problem

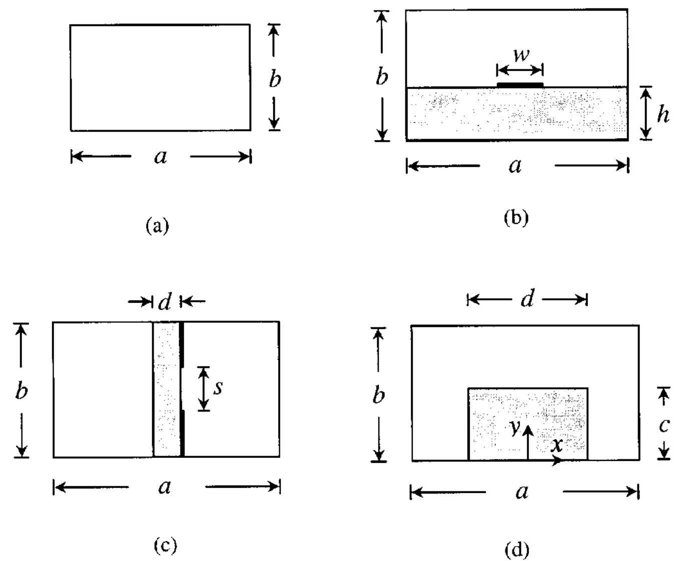

Fig. 3. Typical guided wave structures. (a) Rectangular waveguide. (b) Boxed microstrip line with anisotropic substrate. (c) Unilateral fin-line. (d) Partially filled inhomogeneous dielectric loaded waveguide.

$$[A] \cdot \{x\} = \lambda [B] \cdot \{x\} \tag{14}$$

where $[B]$ is a unit matrix. Since the eigenvalue λ of interest will not equal zero, the zero boundary condition of the ith element of vector $\{x\}$ can be implemented by multiplying a very large number, for example, 10^{20} to the matrix elements a_{ii} and b_{ii} such that the ith equation is dominated by $10^{20}(a_{ii} - \lambda b_{ii})x_i = 0$ or $x_i = 0$.

III. Numerical Results

To verify the proposed method, several typical guided wave structures are analyzed. As the first example, an empty rectangular waveguide with width $a = 19.05$ mm and height $b = 9.525$ mm, as shown in Fig. 3(a), is analyzed for not only the fundamental mode but also the first four higher order modes. The calculated numerical results are compared with the exact solution. As shown in Fig. 4(a), good agreement is obtained. To validate the method for the cutoff modes, the first three modes in the rectangular waveguide are also calculated when they are under cutoff and the results are presented in Fig. 4(b) along with the exact solutions.

The second case is a boxed microstrip line with electric anisotropic substrate, as shown in Fig. 3(b). The width and the height of the metallic box are $a = 6.5$ mm and $b = 3.5$ mm, respectively. The dielectric diagonal constant tensor of the substrate is given by $\varepsilon_{xx} = \varepsilon_{zz} = 9.4$, and $\varepsilon_{yy} = 11.6$, and its thickness $h = 1.5$ mm. The width of the strip $w = 1.5$ mm and the thickness is negligible. The computed result is superposed with that calculated by FDTD in [4] in Fig. 5 with good correlation.

The third example is an unilateral fin-line with the geometry shown in Fig. 3(c), in which $a = 20$ mm, $b = 10$ mm, $d = 1$ mm, and $s = 4$ mm. The relative permittivity of the dielectric

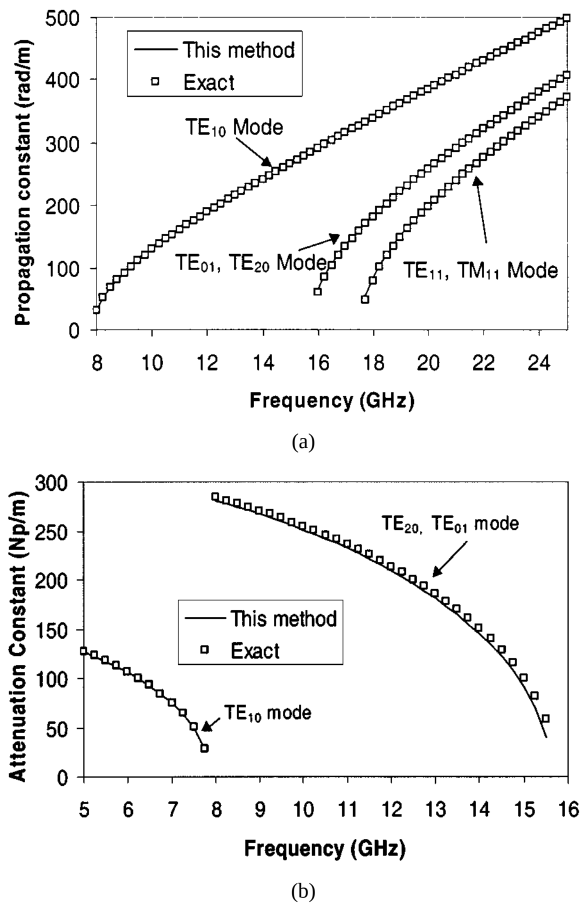

Fig. 4. Dispersion characteristics of a rectangular waveguide. (a) Propagation constant. (b) Attenuation constant.

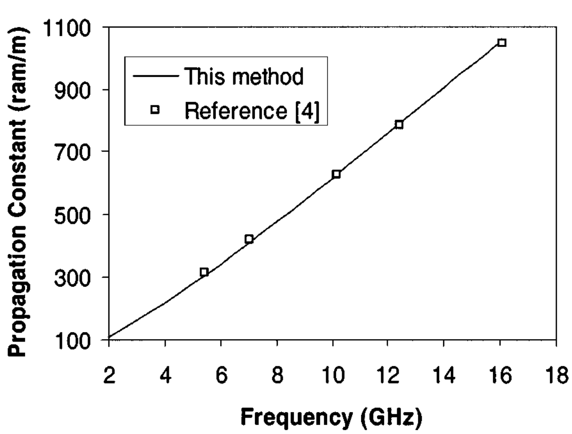

Fig. 5. Dispersion characteristics of a boxed microstrip line with anisotropic substrate.

slab is $\varepsilon_r = 2.22$. The result calculated by the proposed method and that obtained in [3] are superimposed in Fig. 6.

Finally, a partially inhomogeneous dielectric loaded waveguide as shown in Fig. 3(d) is analyzed. The dimensions of the structure are $a = 10.16$ mm, $b = 5.588$ mm, $c = 3.048$ mm, and $d = 5.08$ mm. The relative permittivity of the slab is $\varepsilon_r = 8$. The dispersion curves of the first two modes are obtained and are compared with that of Ansoft-HFSS. As shown in Fig. 7, excellent agreement can be observed.

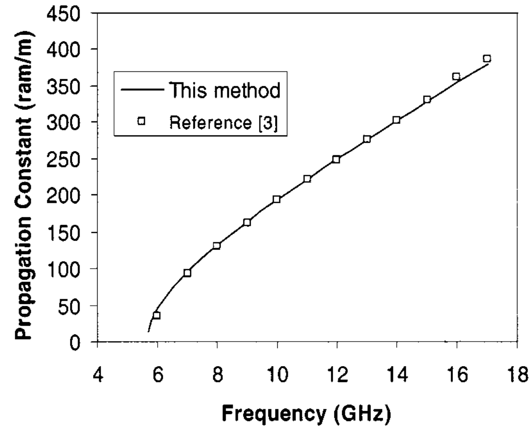

Fig. 6. Dispersion characteristics of a unilateral fin-line.

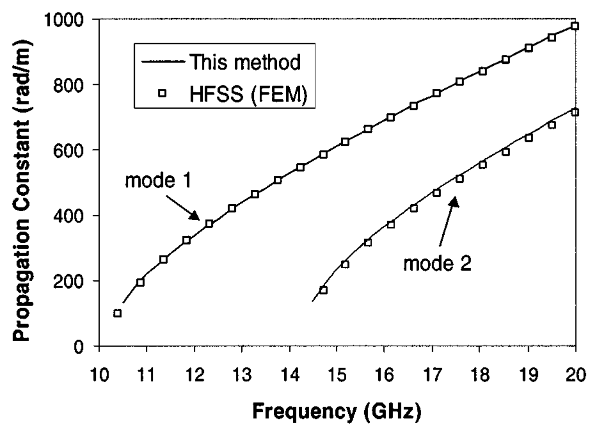

Fig. 7. Dispersion characteristics of a partially filled inhomogeneous dielectric loaded waveguide.

TABLE I
COMPARISON OF THIS METHOD AND SIX-COMPONENT METHOD

		This Method	The Six-component Method
Number of non-zero elements	waveguide	2412	1972
	microstrip	2520	2083
CPU time (sec.)	waveguide	6.04	16.76
	microstrip	15.65	59.49

It is worthwhile mentioning that the example of inhomogeneous dielectric loaded waveguide in [8] is also calculated with this method. For the sake of briefness, the result is not presented here. Excellent agreement has been obtained between the result of this method and that of biorthonormal-basis method, which falls in a totally different category of numerical method.

Although the elimination of longitudinal components does not help to reduce the memory requirement due to the number of terms in remaining equations increases, the CPU time is considerably reduced as compared to the case in which six field components are comprised. The comparison of efficiency in calculating dispersion curves of dominant mode in rectangular waveguide [Fig. 3(a)] and boxed microstrip line [Fig. 3(b)] is shown in Table I. The calculations are carried out on a Pentium III 800 computer using Matlab 6.0.

IV. Conclusion

In this paper, a novel compact 2-D full-wave FDFD method has been proposed for the analysis of the dispersion characteristics of a general guided wave structure. In the algorithm, only four transverse field components are employed in the final resulting eigen equation. Therefore, it is more efficient than the existing approaches involving six field components. Furthermore, since the algorithm determines β for a given k_0 (frequency), it is significantly more attractive and promising in practical applications than the existing methods that inversely seek k_0 for a given β. Additionally, it has been verified that this method also works very well for the modes under cutoff.

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